Chapter 1

Expressions, Equations, and Functions

The study of expressions, equations, and functions is the basis of mathematics. Each mathematical subject requires knowledge of manipulating equations to solve for a variable. Careers such as automobile accident investigators, quality control engineers, and insurance originators use equations to determine the value of variables.

Functions are methods of explaining relationships and can be represented as a rule, a graph, a table, or in words. The amount of money in a savings account, how many miles run in a year, or the number of trout in a pond are all described using functions.

Throughout this chapter, you will learn how to choose the best variables to describe a situation, simplify an expression using the Order of Operations, describe functions in various ways, write equations, and solve problems using a systematic approach.

1.1 Variable Expressions

Who Speaks Math, Anyway?

When someone is having trouble with algebra, they may say, "I don't speak math!" While this may seem weird to you, it is a true statement. Math, like English, French, Spanish, or Arabic, is a secondary language that you must learn in order to be successful. There are verbs and nouns in math, just like in any other language. In order to understand math, you must practice the language.

A verb is a "doing" word, such as running, jumps, or drives. In mathematics, verbs are also "doing" words. A math verb is called an **operation.** Operations can be something you have used before, such as addition, multiplication, subtraction, or division. They can also be much more complex like an exponent or square root.

Example: Suppose you have a job earning \$8.15 per hour. What could you use to quickly find out how much money you would earn for different hours of work?

Solution: You could make a list of all the possible hours, but that would take forever! So instead, you let the "hours you work" be replaced with a symbol, like h for hours, and write an equation such as:

$$amount\ of\ money = 8.15(h)$$

A noun is usually described as a person, place, or thing. In mathematics, nouns are called numbers and **variables.** A variable is a symbol, usually an English letter, written to replace an unknown or changing quantity.

Example: What variables would be choices for the following situations?

a. the number of cars on a road

b. time in minutes of a ball bounce

c. distance from an object

Solution: There are many options, but here are a few to think about.

a. Cars is the changing value, so c is a good choice.

b. Time is the changing value, so t is a good choice.

c. Distance is the varying quantity, so d is a good choice.

Why Do They Do That?

Just like in the English language, mathematics uses several words to describe one thing. For example, *sum, addition, more than,* and *plus* all mean to add numbers together. The following definition shows an example of this.

Definition: To **evaluate** means to follow the verbs in the math sentence. **Evaluate** can also be called simplify or answer.

To begin to evaluate a mathematical **expression**, you must first **substitute** a number for the variable.

Definition: To **substitute** means to replace the variable in the sentence with a value.

Now try out your new vocabulary.

Example: EVALUATE $7y - 11$, when $y = 4$.

Solution: Evaluate means to follow the directions, which is to take 7 times y and subtract 11. Because y is the number 4,

$7 \times 4 - 11$	We have "substituted" the number 4 for y.
$28 - 11$	Multiplying 7 and 4
17	Subtracting 11 from 28
The solution is 17.	

Because algebra uses variables to represent the unknown quantities, the multiplication symbol $\times$ is often confused with the variable x. To help avoid confusion, mathematicians replace the multiplication symbol with parentheses ( ), the multiplication dot $\cdot$, or by writing the expressions side by side.

Example: Rewrite $P = 2 \times l + 2 \times w$ with alternative multiplication symbols.

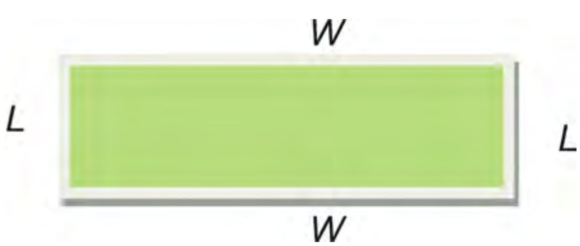
Solution: $P = 2 \times l + 2 \times w$ can be written as $P = 2 \cdot l + 2 \cdot w$

It can also be written as $P = 2l + 2w$.

The following is a real-life example that shows the importance of evaluating a mathematical variable.

Example: To prevent major accidents or injuries, these horses must be fenced in a rectangular pasture. If the dimensions of the pasture are 300 feet by 225 feet, how much fencing should the ranch hand purchase to enclose the pasture?

Solution: Begin by drawing a diagram of the pasture and labeling what you know.



To find the amount of fencing needed, you must add all the sides together;

$$L + L + W + W.$$

By substituting the dimensions of the pasture for the variables L and W, the expression becomes

$$300 + 300 + 225 + 225.$$

Now we must evaluate by adding the values together. The ranch hand must purchase 1,050 feet of fencing.

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set.However, the practice exercise is the same in both. CK-12 Basic Algebra: Variable Expressions (12:26) In 1 – 4, write the expression in a more

Figure 1.1: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/479

condensed form by leaving out a multiplication symbol.

1. $2 \times 11x$
2. $1.35 \cdot y$
3. $3 \times \frac{1}{4}$
4. $\frac{1}{4} \cdot z$

In 5 – 9, evaluate the expression.

5. $5m + 7 \textit{ when } m = 3$.
6. $\frac{1}{3}(c) \textit{ when } c = 63$.
7. $\$8.15(h) \textit{ when } h = 40$.
8. $(k - 11) \div 8 \textit{ when } k = 43$.
9. Evaluate $(-2)^2 + 3(j) \textit{ when } j = -3$.

In 10 – 17, evaluate the expressions. Let $a = -3, \ b = 2, \ c = 5, \textit{ and } d = -4$.

10. $2a + 3b$

11. $4c+d$
12. $5ac-2b$
13. $\frac{2a}{c-d}$
14. $\frac{3b}{d}$
15. $\frac{a-4b}{3c+2d}$
16. $\frac{1}{a+b}$
17. $\frac{ab}{cd}$

In 18 – 25, evaluate the expressions. Let $x=-1,\ y=2,\ z=-3$, and $w=4$.

18. $8x^3$
19. $\frac{5x^2}{6z^3}$
20. $3z^2-5w^2$
21. x^2-y^2
22. $\frac{z^3+w^3}{z^3-w^3}$
23. $2x^2-3x^2+5x-4$
24. $4w^3+3w^2-w+2$
25. $3+\frac{1}{z^2}$

In 26 – 30, choose an appropriate variable to describe each situation.

26. The number of hours you work in a week
27. The distance you travel
28. The height of an object over time
29. The area of a square
30. The number of steps you take in a minute

In 31 – 35, underline the math verb(s) in the sentence.

31. The product of six and v
32. Four plus y minus six
33. Sixteen squared
34. U divided by 3 minus eight
35. The square root of 225

In 36 – 40, evaluate the real-life problems.

36. The measurement around the widest part of these holiday bulbs is called their *circumference.* The formula for circumference is $2(r)\pi$, where $\pi \approx 3.14$ and r is the radius of the circle. Suppose the radius is 1.25 inches. Find the *circumference.*
37. The dimensions of a piece of notebook paper are 8.5 inches by 11 inches. Evaluate the writing area of the paper. The formula for area is length $\times$ width.
38. Sonya purchases 16 cans of soda at \$0.99 each. What is the amount Sonya spent on soda?
39. Mia works at a job earning \$4.75 per hour. How many hours should she work to earn \$124.00?
40. The area of a square is the side length squared. Evaluate the area of a square with side length 10.5 miles.

Figure 1.2: Christmas Baubles by Petr Kratochvil

1.2 Order of Operations

The Mystery of Math Verbs

Some math verbs are "stronger" than others and must be done first. This method is known as the **Order of Operations.**

A mnemonic (a saying that helps you remember something difficult) for the **Order of Operations** is PEMDAS - Please Excuse My Daring Aunt Sophie.

The Order of Operations:

Whatever is found inside **PARENTHESES** must be done first. **EXPONENTS** are to be simplified next. **MULTIPLICATION** and **DIVISION** are equally important and must be performed moving left to right. **ADDITION** and **SUBTRACTION** are also equally important and must be performed moving left to right.

Example 1: Use the Order of Operations to simplify $(7 - 2) \times 4 \div 2 - 3$

Solution: First, we check for parentheses. Yes, there they are and must be done first.

$$(7 - 2) \times 4 \div 2 - 3 = (5) \times 4 \div 2 - 3$$

Next we look for exponents (little numbers written a little above the others). No, there are no exponents so we skip to the next math verb.

Multiplication and division are equally important and must be done from left to right.

$$5 \times 4 \div 2 - 3 = 20 \div 2 - 3$$
$$20 \div 2 - 3 = 10 - 3$$

Finally, addition and subtraction are equally important and must be done from left to right.

$10 - 3 = 7$ This is our answer.

Example 2: Use the Order of Operations to simplify the following expressions.

a) $3 \times 5 - 7 \div 2$

b) $3 \times (5 - 7) \div 2$

c) $(3 \times 5) - (7 \div 2)$

Solutions:

a) There are no parentheses and no exponents. Go directly to multiplication and division from left to right: $3 \times 5 - 7 \div 2 = 15 - 7 \div 2 = 15 - 3.5$

Now subtract: $15 - 3.5 = 11.5$

b) Parentheses must be done first: $3 \times (-2) \div 2$

There are no exponents, so multiplication and division come next and are done left to right: $3 \times (-2) \div 2 = -6 \div 2 = -3$

c) Parentheses must be done first: $(3 \times 5) - (7 \div 2) = 15 - 3.5$

There are no exponents, multiplication, division, or addition, so simplify:

$$15 - 3.5 = 11.5$$

Parentheses are used two ways. The first is to alter the Order of Operations in a given expression, such as example (b). The second way is to clarify an expression, making it easier to understand.

Some expressions contain no parentheses while others contain several sets of parentheses. Some expressions even have parentheses inside parentheses! When faced with **nested parentheses**, start at the innermost parentheses and work outward.

Example 3: Use the Order of Operations to simplify $8 - [19 - (2 + 5) - 7]$

Solution: Begin with the innermost parentheses:

$$8 - [19 - (2 + 5) - 7] = 8 - [19 - 7 - 7]$$

Simplify according to the Order of Operations:

$$8 - [19 - 7 - 7] = 8 - [5] = 3$$

Evaluating Algebraic Expressions with Fraction Bars

Fraction bars count as grouping symbols for PEMDAS, and should be treated as a set of parentheses. All numerators and all denominators can be treated as if they have invisible parentheses. When **real** parentheses are also present, remember that the innermost grouping symbols should be evaluated first. If, for example, parentheses appear on a numerator, they would take precedence over the fraction bar. If the parentheses appear outside of the fraction, then the fraction bar takes precedence.

Example 4: Use the Order of Operations to simplify the following expressions.

a) $\frac{z+3}{4} - 1 \ when \ z = 2$

b) $\left(\frac{a+2}{b+4} - 1\right) + b \ when \ a = 3$ and $b = 1$

c) $2 \times \left(\frac{w+(x-2z)}{(y+2)^2} - 1\right) \ when \ w = 11, \ x = 3, \ y = 1$ and $z = -2$

Solutions: Begin each expression by substituting the appropriate value for the variable:

a) $\frac{(2+3)}{4} - 1 = \frac{5}{4} - 1$. Rewriting 1 as a fraction, the expression becomes:

$$\frac{5}{4} - \frac{4}{4} = \frac{1}{4}$$

b) $\frac{(3+2)}{(1+4)} = \frac{5}{5} = 1$

$(1-1)+b$ Substituting 1 for b, the expression becomes $0+1=1$

c) $2\left(\frac{[11+(3-2(-2))]}{[(1+2)^2)]} - 1\right) = 2\left(\frac{(11+7)}{3^2} - 1\right) = 2\left(\frac{18}{9} - 1\right)$

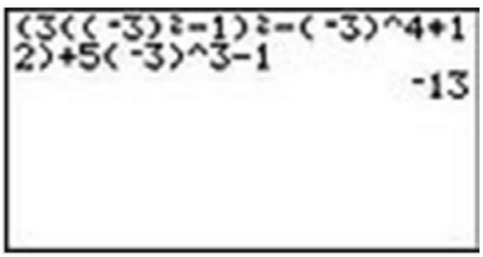
Continue simplifying: $2\left(\frac{18}{9} - \frac{9}{9}\right) = 2\left(\frac{9}{9}\right) = 2(1) = 2$

Using a Calculator to Evaluate Algebraic Expressions

A calculator, especially a graphing calculator, is a very useful tool in evaluating algebraic expressions. The graphing calculator follows the Order of Operations, PEMDAS. In this section, we will explain two ways of evaluating expressions with the graphing calculator.

Method #1: This method is the direct input method. After substituting all values for the variables, you type in the expression, symbol for symbol, into your calculator.

Evaluate $[3(x^2-1)^2 - x^4 + 12] + 5x^3 - 1$ *when* $x = -3$.



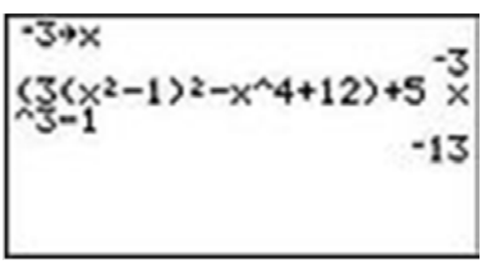
Substitute the value $x = -3$ *into the expression.*

$$[3((-3)^2-1)^2 - (-3)^4 + 12] + 5(-3)^3 - 1$$

The potential error here is that you may forget a sign or a set of parentheses, especially if the expression is long or complicated. Make sure you check your input before writing your answer. An alternative is to type the expression in by appropriate chunks – do one set of parentheses, then another, and so on.

Method #2: This method uses the STORE function of the Texas Instrument graphing calculators, such as the TI-83, TI-84, or TI-84 Plus.

First, store the value $x = -3$ *in the calculator. Type* -3 **[STO]** x. (*The letter* x *can be entered using the* x-**[VAR]** *button or* **[ALPHA]** + **[STO]**). *Then type in the expression in the calculator and press* **[ENTER]**.

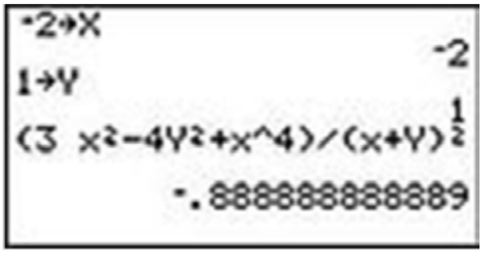


The answer is -13.

Note: On graphing calculators there is a difference between the minus sign and the negative sign. When we stored the value negative three, we needed to use the negative sign, which is to the left of the **[ENTER]** button on the calculator. On the other hand, to perform the subtraction operation in the expression we used the minus sign. The minus sign is right above the plus sign on the right.

You can also use a graphing calculator to evaluate expressions with more than one variable.

Evaluate the expression: $\frac{3x^2-4y^2+x^4}{(x+y)^{\frac{1}{2}}}$ *for* $x=-2, y=1$.



Store the values of x and y. -2 **[STO]** x, 1 **[STO]** y. The letters x and y can be entered using **[ALPHA]** + **[KEY]**. Input the expression in the calculator. When an expression shows the division of two expressions be sure to use parentheses: (numerator) $\div$ (denominator). Press **[ENTER]** to obtain the answer $-.8\bar{8}$ or $-\frac{8}{9}$.

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set.However, the practice exercise is the same in both. CK-12 Basic Algebra: Order of Operations (14:23)

Figure 1.3: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/708

Use the Order of Operations to simplify the following expressions.

1. $8-(19-(2+5)-7)$
2. $2+7\times 11-12\div 3$
3. $(3+7)\div(7-12)$
4. $\frac{2\cdot(3+(2-1))}{4-(6+2)}-(3-5)$
5. $8\cdot 5+6^2$
6. $9\div 3\times 7-2^3+7$
7. $8+12\div 6+6$
8. $(7^2-3^2)\div 8$

Evaluate the following expressions involving variables.

9. $\frac{jk}{j+k}$ when $j=6$ and $k=12$.
10. $2y^2$ when $x=1$ and $y=5$

11. $3x^2+2x+1$ when $x=5$
12. $(y^2-x)^2$ when $x=2$ and $y=1$

Evaluate the following expressions involving variables.

13. $\frac{4x}{9x^2-3x+1}$ when $x=2$
14. $\frac{z^2}{x+y}+\frac{x^2}{x-y}$ when $x=1,\ y=-2$, and $z=4$.
15. $\frac{4xyz}{y^2-x^2}$ when $x=3,\ y=2$, and $z=5$
16. $\frac{x^2-z^2}{xz-2x(z-x)}$ when $x=-1$ and $z=3$

The formula to find the volume of a square pyramid is $V=\frac{s^2(h)}{3}$. Evaluate the volume for the given values.

17. $s=4\ inches, h=18\ inches$
18. $s=10\ feet, h=50\ feet$
19. $h=7\ meters, s=12\ meters$
20. $h=27\ feet, s=13\ feet$
21. $s=16\ cm, h=90\ cm$

In 22 – 25, insert parentheses in each expression to make a true equation.

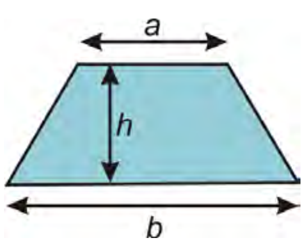
22. $5-2\cdot 6-4+2=5$
23. $12\div 4+10-3\cdot 3+7=11$
24. $22-32-5\cdot 3-6=30$
25. $12-8-4\cdot 5=-8$

In 26 – 29, evaluate each expression using a graphing calculator.

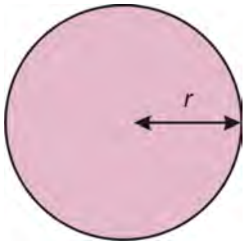
26. $x^2+2x-xy$ when $x=250$ and $y=-120$
27. $(xy-y^4)^2$ when $x=0.02$ and $y=-0.025$
28. $\frac{x+y-z}{xy+yz+xz}$ when $x=\frac{1}{2},\ y=\frac{3}{2}$, and $z=-1$
29. $\frac{(x+y)^2}{4x^2-y^2}$ when $x=3$ and $y=-5d$
30. The formula to find the volume of a spherical object (like a ball) is $V=\frac{4}{3}(\pi)r^3$, where $r=$ the radius of the sphere. Determine the volume for a grapefruit with a radius of 9 cm.

Mixed Review

31. Let $x=-1$. Find the value of $-9x+2$.
32. The area of a trapezoid is given by the equation $A=\frac{h}{2}(a+b)$. Find the area of a trapezoid with bases $a=10\ cm, b=15\ cm$, and height $h=8\ cm$.



33. The area of a circle is given by the formula $A = \pi r^2$. Find the area of a circle with radius $r = 17$ inches.



1.3 Patterns and Expressions

In mathematics, especially in algebra, we look for patterns in the numbers that we see. Using mathematical verbs and variables studied in lessons 1.1 and 1.2, expressions can be written to describe a pattern.

Definition: An **algebraic expression** is a mathematical phrase combining numbers and/or variables using mathematical operations.

Consider a theme park charging an admission of \$28 per person. A rule can be written to describe the relationship between the amount of money taken at the ticket booth and the number of people entering the park. In words, the relationship can be stated as "*The money taken in dollars is (equals) twenty-eight times the number of people who enter the park.*"

The English phrase above can be translated (written in another language) into an algebraic expression. Using mathematical verbs and nouns learned from previous lessons, any sentence can be written as an algebraic expression.

Example 1: Write an algebraic expression for the following phrase.

The product of c and 4.

Solution: The verb is *product*, meaning "to multiply." Therefore, the phrase is asking for the answer found by multiplying c and 4. The nouns are the number 4 and the variable c. The expression becomes $4 \times c$, $4(c)$, or using shorthand, $4c$.

Example 2: Write an expression to describe the amount of revenue of the theme park.

Solution: An appropriate variable to describe the number of people could be p. Rewriting the English phrase into a mathematical phrase, it becomes $28 \times p$.

Using Words to Describe Patterns

Sometimes patterns are given in *tabular* format (meaning presented in a table). An important job of analysts is to describe a pattern so others can understand it.

Example 3: Using the table below, describe the pattern in words.

x	-1	0	1	2	3	4
y	-5	0	5	10	15	20

Solution: We can see from the table that y is five times bigger than x. Therefore, the pattern is that the "y value is five times larger than the x value."

Example 4: Using the table below, describe the pattern in words and in an expression.

Zarina has a $100 gift card and has been spending money in small regular amounts. She checks the balance on the card weekly, and records the balance in the following table.

Table 1.1:

Week #	**Balance ($)**
1	100
2	78
3	56
4	34

Solution: Each week the amount of her gift card is $22 less than the week before. The pattern in words is: *"The gift card started at $100 and is decreasing by $22 each week."*

The expression found in example 4 can be used to answer many situations. Suppose, for instance, that Zarina has been using her gift card for 4 weeks. By substituting the number 4 for the variable w, it can be determined that Zarina has $12 left on her gift card.

Solution:

$$100 - 22w$$

When $w = 4$, the expression becomes

$$\begin{aligned}&100 - 22(4)\\&100 - 88\\&12\end{aligned}$$

After 4 weeks, Zarina has $12 left on her gift card.

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set.However, the practice exercise is the same in both. CK-12 Basic Algebra: Patterns and Equations (13:18)

Figure 1.4: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/709

For exercises 1 – 15, translate the English phrase into an algebraic expression. For the exercises without a stated variable, choose a letter to represent the unknown quantity.

1. Sixteen more than a number
2. The quotient of h and 8
3. Forty-two less than y
4. The product of k and three
5. The sum of g and -7
6. r minus 5.8
7. 6 more than 5 times a number
8. 6 divided by a number minus 12
9. A number divided by -11
10. 27 less than a number times four
11. The quotient of 9.6 and m
12. 2 less than 10 times a number
13. The quotient of d and five times s
14. 35 less than x
15. The product of 6, -9, and u

In exercises 16 – 24, write an English phrase for each algebraic expression

16. $J - 9$
17. $\frac{n}{14}$
18. $17 - a$
19. $3l - 16$
20. $\frac{1}{2}(h)(b)$
21. $\frac{b}{3} + \frac{z}{2}$
22. $4.7 - 2f$
23. $5.8 + k$
24. $2l + 2w$

In exercises 25 – 28, define a variable to represent the unknown quantity and write an expression to describe the situation.

25. The *unit cost* represents the quotient of the total cost and number of items purchased. Write an expression to represent the unit cost of the following: The total cost is \$14.50 for n objects.
26. The area of a square is the side length squared.
27. The total length of ribbon needed to make dance outfits is 15 times the number of outfits.

28. What is the remaining amount of chocolate squares if you started with 16 and have eaten some?
29. Describe a real-world situation that can be represented by $h + 9$.
30. What is the difference between $\frac{7}{m}$ and $\frac{m}{7}$?

In questions 31 – 34, write the pattern of the table: a) in words and b) with an algebraic expression.

31. Number of workers and number of video games packaged

People	0	1	2	5	10	50	200
Amount	0	65	87	109	131	153	175

32. The number of hours worked and the total pay

Hours	1	2	3	4	5	6
Total Pay	15	22	29	36	43	50

33. The number of hours of an experiment and the total number of bacteria

Hours	0	1	2	5	10
Bacteria	0	2	4	32	1024

34. With each filled seat, the number of people on a Ferris wheel doubles.
    (a) Write an expression to describe this situation.
    (b) How many people are on a Ferris wheel with 17 seats filled?
35. Using the theme park situation from the lesson, how much revenue would be generated by 2,518 people?

Mixed Review

36. Use parentheses to make the equation true: $10 + 6 \div 2 - 3 = 5$.
37. Find the value of $5x^2 - 4y$ for $x = -4$ and $y = 5$.
38. Find the value of $\frac{x^2y^3}{x^3+y^2}$ for $x = 2$ and $y = -4$.
39. Simplify: $2 - (t - 7)^2 \times (u^3 - v)$ when $t = 19, u = 4$, and $v = 2$.
40. Simplify: $2 - (19 - 7)^2 \times (4^3 - 2)$.

1.4 Equations and Inequalities

When an algebraic expression is set equal to another value, variable, or expression, a new mathematical sentence is created. This sentence is called an **equation.**

Definition: An **algebraic equation** is a mathematical sentence connecting an expression to a value, a variable, or another expression with an equal sign (=).

Consider the theme park situation from lesson 1.3. Suppose there is a concession stand selling burgers and French fries. Each burger costs \$2.50 and each order of French fries costs \$1.75. You and your family will spend exactly \$25.00 on food. How many burgers can be purchased? How many orders of fries? How many of each type can be purchased if your family plans to buy a combination of burgers and fries?

The underlined word exactly lends a clue to the type of mathematical sentence you will need to write to model this situation.

These words can be used to symbolize the equal sign:

Exactly, equivalent, the same as, identical, is

The word *exactly* is synonymous with equal, so this word is directing us to write an equation. Using the methods learned in lessons 1.2 and 1.3, read every word in the sentence and translate each into mathematical symbols.

Example 1: Your family is planning to purchase only burgers. How many can be purchased with \$25.00?

Solution:

Step 1: Choose a variable to represent the unknown quantity, say b for burgers.

Step 2: Write an equation to represent the situation: $2.50b = 25.00$.

Step 3: Think. What number multiplied by 2.50 equals 25.00?

The solution is 10, so your family can purchase exactly ten burgers.

Example 2: Translate the following into equations:

a) *9 less than twice a number is* 33.

b) *Five more than four times a number is* 21.

c) \$20.00 *was one-quarter of the money spent on pizza.*

Solutions:

a) Let "a number" be n. So, twice a number is $2n$.

Nine less than that is $2n - 9$.

The word *is* means the equal sign, so $2n - 9 = 33$.

b) Let "a number" be x. So five more than four times a number is 21 can be written as: $4x + 5 = 21$.

c) Let "of the money" be m. The equation could be written as $\frac{1}{4}m = 20.00$.

Definition: The **solution** to an equation or inequality is the value (or multiple values) that make the equation or inequality true.

Using statement (c) from example 2, find the solution.

$$\frac{1}{4}m = 20.00$$

Think: One-quarter can also be thought of as *divide by four.* What divided by 4 equals 20.00?

The solution is 80. So, the money spent on pizza was \$80.00.

Checking an answer to an equation is almost as important as the equation itself. By substituting the value for the variable, you are making sure both sides of the equation balance.

Example 3: Check that $x = 5$ is the solution to the equation $3x + 2 = -2x + 27$.

Solution: To check that $x = 5$ is the solution to the equation, substitute the value of 5 for the variable, x:

$$\begin{aligned}
3x + 2 &= -2x + 27 \\
3 \cdot x + 2 &= -2 \cdot x + 27 \\
3 \cdot 5 + 2 &= -2 \cdot 5 + 27 \\
15 + 2 &= -10 + 27 \\
17 &= 17
\end{aligned}$$

Because $17 = 17$ is a true statement, we can conclude that $x = 5$ is a solution to $3x + 2 = -2x + 27$.

Example 4: Is $z = 3$ a solution to $z^2 + 2z = 8$?

Solution: Begin by substituting the value of 3 for z.

$$\begin{aligned}
3^2 + 2(3) &= 8 \\
9 + 6 &= 8 \\
15 &= 8
\end{aligned}$$

Because $15 = 8$ is NOT a true statement, we can conclude that $z = 3$ is not a solution to $z^2 + 2z = 8$.

Sometimes Things Are Not Equal

In some cases there are multiple answers to a problem or the situation requires something that is not exactly equal to another value. When a mathematical sentence involves something other than an equal sign, an **inequality** is formed.

Definition: An **algebraic inequality** is a mathematical sentence connecting an expression to a value, a variable, or another expression with an inequality sign.

Listed below are the most common inequality signs.

$>$ "greater than"

$\geq$ "greater than or equal to"

$\leq$ "less than or equal to"

$<$ "less than"

$\neq$ "not equal to"

Below are several examples of inequalities.

$$3x < 5 \qquad x^2 + 2x - 1 > 0 \qquad \frac{3x}{4} \geq \frac{x}{2} - 3 \qquad 4 - x \leq 2x$$

Example 5: Translate the following into an inequality: Avocados cost \$1.59 per pound. How many pounds of avocados can be purchased for less than \$7.00?

Solution: Choose a variable to represent the number of pounds of avocados purchased, say a.

$$1.59(a) < 7$$

You will be asked to solve this inequality in the exercises

Checking the Solution to an Inequality

Unlike equations, inequalities typically have more than one solution. Checking solutions to inequalities is more complex than checking solutions to equations. The key to checking a solution to an inequality is to choose a number that occurs within the solution set.

Example 6: Check that $m \leq 10$ is a solution to $4m + 30 \leq 70$.

Solution: If the solution set is true, any value less than or equal to 10 should make the original inequality true.

Choose a value less than 10, say 4. Substitute this value for the variable m.

$$\begin{aligned}&4(4) + 30\\&16 + 30\\&46 \leq 70\end{aligned}$$

The value found when $m = 4$ is less than 70. Therefore, the solution set is true.

Why was the value 10 not chosen? Endpoints are not chosen when checking an inequality because the direction of the inequality needs to be tested. Special care needs to be taken when checking the solutions to an inequality.

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set.However, the practice exercise is the same in both. CK-12 Basic Algebra: Equations and Inequalities (16:11)

Figure 1.5: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/712

1. Define *solution*.
2. What is the difference between an algebraic equation and an algebraic inequality? Give an example of each.
3. What are the five most common inequality symbols?

In 4 – 11, define the variables and translate the following statements into algebraic equations.

4. Peter's Lawn Mowing Service charges \$10 per job and \$0.20 per square yard. Peter earns \$25 for a job.
5. Renting the ice-skating rink for a birthday party costs \$200 plus \$4 per person. The rental costs \$324 in total.
6. Renting a car costs \$55 per day plus \$0.45 per mile. The cost of the rental is \$100.
7. Nadia gave Peter 4 more blocks than he already had. He already had 7 blocks.
8. A bus can seat 65 passengers or fewer.
9. The sum of two consecutive integers is less than 54.
10. An amount of money is invested at 5% annual interest. The interest earned at the end of the year is greater than or equal to \$250.
11. You buy hamburgers at a fast food restaurant. A hamburger costs \$0.49. You have at most \$3 to spend. Write an inequality for the number of hamburgers you can buy.

In 12 – 15, check that the given number is a solution to the corresponding equation.

12. $a = -3;\ 4a + 3 = -9$
13. $x = \frac{4}{3};\ \frac{3}{4}x + \frac{1}{2} = \frac{3}{2}$
14. $y = 2;\ 2.5y - 10.0 = -5.0$
15. $z = -5;\ 2(5 - 2z) = 20 - 2(z - 1)$

For exercises 16 – 19, check that the given number is a solution to the corresponding inequality.

16. $x = 12;\ 2(x + 6) \leq 8x$

17. $z = -9;\ 1.4z + 5.2 > 0.4z$
18. $y = 40;\ -\frac{5}{2}y + \frac{1}{2} < -18$
19. $t = 0.4;\ 80 \geq 10(3t + 2)$

In 20 – 24, find the value of the variable.

20. $m + 3 = 10$
21. $6 \times k = 96$
22. $9 - f = 1$
23. $8h = 808$
24. $a + 348 = 0$
25. Using the burger and French fries situation from the lesson, give three combinations of burgers and fries your family can buy without spending more than \$25.00.
26. Solve the avocado inequality from Example 5 and check your solution.
27. You are having a party and are making sliders. Each person will eat 5 sliders. There will be seven people at your party. How many sliders do you need to make?
28. The cost of a Ford Focus is 27% of the price of a Lexus GS 450h. If the price of the Ford is \$15,000, what is the price of the Lexus?
29. On your new job you can be paid in one of two ways. You can either be paid \$1000 per month plus 6% commission on total sales or be paid \$1200 per month plus 5% commission on sales over \$2000. For what amount of sales is the first option better than the second option? Assume there are always sales over \$2000.
30. Suppose your family will purchase only orders of French fries using the information found in the opener of this lesson. How many orders of fries can be purchased for \$25.00?

Mixed Review

31. Translate into an algebraic equation: 17 less than a number is 65.
32. Simplify the expression: $3^4 \div (9 \times 3) + 6 - 2$.
33. Rewrite the following without the multiplication sign: $A = \frac{1}{2} \cdot b \cdot h$.
34. The volume of a box without a lid is given by the formula $V = 4x(10 - x)^2$, where x is a length in inches and V is the volume in cubic inches. What is the volume of the box when $x = 2$?

1.5 Functions as Rules and Tables

Instead of purchasing a one-day ticket to the theme park, Joseph decided to pay by ride. Each ride costs \$2.00. To describe the amount of money Joseph will spend, several mathematical concepts can be used.

First, an expression can be written to describe the relationship between the cost per ride and the number of rides, r. An equation can also be written if the total amount he wants to spend is known. An inequality can be used if Joseph wanted to spend less than a certain amount.

Example 1: Using Joseph's situation, write the following:

a. An expression representing his total amount spent

b. An equation that shows Joseph wants to spend exactly \$22.00 on rides

c. An inequality that describes the fact that Joseph will not spend more than \$26.00 on rides

Solution: The variable in this situation is the number of rides Joseph will pay for. Call this r.

a. $2(r)$

b. $2(r) = 22$

c. $2(r) \leq 26$

In addition to an expression, equation, or inequality, Joseph's situation can be expressed in the form of a function or a table.

Definition: A **function** is a relationship between two variables such that the input value has ONLY one output value.

Writing Equations as Functions

A function is a set of ordered pairs in which the first coordinate, usually x, matches with exactly one second coordinate, y. Equations that follow this definition can be written in function notation. The y coordinate represents the **dependent variable**, meaning the values of this variable depend upon what is substituted for the other variable.

Consider Joseph's equation $m = 2r$. Using function notation, the value of the equation (the money spent m) is replaced with $f(r)$. f represents the function name and (r) represents the variable. In this case the parentheses do not mean multiplication; they separate the function name from the **independent variable.**

$$\overset{\substack{input \\ \downarrow}}{\underbrace{f(x)}_{\substack{function \\ box}}} = y \leftarrow output$$

Example 2: Rewrite the following equations in function notation.

a. $y = 7x - 3$

b. $d = 65t$

c. $F = 1.8C + 32$

Solution:

a. According to the definition of a function, $y = f(x)$, so $f(x) = 7x - 3$.

b. This time the dependent variable is d. Function notation replaces the dependent variable, so $d = f(t) = 65t$.

c. $F = f(C) = 1.8C + 32$

Why Use Function Notation?

Why is it necessary to use function notation? The necessity stems from using multiple equations. Function notation allows one to easily decipher between the equations. Suppose Joseph, Lacy, Kevin, and Alfred all went to the theme park together and chose to pay $2.00 for each ride. Each person would have the same equation $m = 2r$. Without asking each friend, we could not tell which equation belonged to whom. By substituting function notation for the dependent variable, it is easy to tell which function belongs to whom. By using function notation, it will be much easier to graph multiple lines (Chapter 4).

Example 3: Write functions to represent the total each friend spent at the park.

Solution: $J(r) = 2r$ represents Joseph's total, $L(r) = 2r$ represents Lacy's total, $K(r) = 2r$ represents Kevin's total, and $A(r) = 2r$ represents Alfred's total.

Using a Function to Generate a Table

A function really is an equation. Therefore, a table of values can be created by choosing values to represent the **independent variable.** The answers to each substitution represent $f(x)$.

Use Joseph's function to generate a table of values. Because the variable represents the number of rides Joseph will pay for, negative values do not make sense and are not included in the value of the independent variable.

Table 1.2:

R	$J(r) = 2r$
0	$2(0) = 0$
1	$2(1) = 2$
2	$2(2) = 4$
3	$2(3) = 6$
4	$2(4) = 8$
5	$2(5) = 10$
6	$2(6) = 12$

As you can see, the list cannot include every possibility. A table allows for precise organization of data. It also provides an easy reference for looking up data and offers a set of coordinate points that can be plotted to create a graphical representation of the function. A table does have limitations; namely it cannot represent infinite amounts of data and it does not always show the possibility of fractional values for the independent variable.

Domain and Range of a Function

The set of all possible input values for the independent variable is called the **domain.** The domain can be expressed in words, as a set, or as an inequality. The values resulting from the substitution of the domain represent the **range** of a function.

The domain of Joseph's situation will not include negative numbers because it does not make sense to ride negative rides. He also cannot ride a fraction of a ride, so decimals and fractional values do not make sense as input values. Therefore, the values of the independent variable r will be whole numbers beginning at zero.

Domain: All whole numbers

The values resulting from the substitution of whole numbers are whole numbers times two. Therefore, the **range** of Joseph's situation is still whole numbers just twice as large.

Range: All even whole numbers

Example 4: A tennis ball is bounced from a height and bounces back to 75% of its previous height. Write its function and determine its domain and range.

Solution: The function of this situation is $h(b) = 0.75b$, where b represents the previous bounce height.

Domain: The previous bounce height can be any positive number, so $b \geq 0$.

Range: The new height is 75% of the previous height, and therefore will also be any positive number (decimal or whole number), so the range is **all positive real numbers.**

Multimedia Link For another look at the domain of a function, see the following video where the narrator solves a sample problem from the California Standards Test about finding the domain of an unusual function. Khan Academy CA Algebra I Functions (6:34)

Figure 1.6: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/88

Write a Function Rule

In many situations, data is collected by conducting a survey or an experiment. To visualize the data, it is arranged into a table. Most often, a function rule is needed to predict additional values of the independent variable.

Example 5: *Write a function rule for the table.*

Number of CDs	2	4	6	8	10
Cost ($)	24	48	72	96	120

Solution: You pay \$24 for 2 CDs, \$48 for 4 CDs, and \$120 for 10 CDs. That means that each CD costs \$12.

We can write the function rule.

Cost = \$12 × number of CDs or $f(x) = 12x$

Example 6: *Write a function rule for the table.*

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	1	2	3

Solution: The values of the dependent variable are always the positive outcomes of the input values. This relationship has a special name, the absolute value. The function rule looks like this: $f(x) = |x|$.

Represent a Real-World Situation with a Function

Let's look at a real-world situation that can be represented by a function.

Example 7: *Maya has an internet service that currently has a monthly access fee of \$11.95 and a connection fee of \$0.50 per hour. Represent her monthly cost as a function of connection time.*

Solution: Let x = the number of hours Maya spends on the internet in one month and let y = Maya's monthly cost. The monthly fee is \$11.95 with an hourly charge of \$0.50.

The total cost = flat fee + hourly fee × number of hours. The function is $y = f(x) = 11.95 + 0.50x$

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set.However, the practice exercise is the same in both. CK-12 Basic Algebra: Domain and Range of a Function (12:52)

Figure 1.7: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/454

1. Rewrite using function notation: $y = \frac{5}{6}x - 2$.
2. What is one benefit of using function notation?
3. Define *domain.*
4. *True or false?* Range is the set of all possible inputs for the independent variable.
5. Generate a table from $-5 \le x \le 5$ for $f(x) = -(x)^2 - 2$
6. Use the following situation for question 6: *Sheri is saving for her first car. She currently has \$515.85 and is savings \$62 each week.*

(a) Write a function rule for the situation.
(b) Can the domain be "all real numbers"? Explain your thinking.
(c) How many weeks would it take Sheri to save $1,795.00?

In 7 - 11, identify the domain and range of the function.

7. Dustin charges $10 per hour for mowing lawns.
8. Maria charges $25 per hour for math tutoring, with a minimum charge of $15.
9. $f(x) = 15x - 12$
10. $f(x) = 2x^2 + 5$
11. $f(x) = \frac{1}{x}$
12. What is the range of the function $y = x^2 - 5$ when the domain is $-2, -1, 0, 1, 2$?
13. What is the range of the function $y = 2x - \frac{3}{4}$ when the domain is $-2.5, 1.5, 5$?
14. Angie makes $6.50 per hour working as a cashier at the grocery store. Make a table of values that shows her earning for the input values 5, 10, 15, 20, 25, 30.
15. The area of a triangle is given by: $A = \frac{1}{2}bh$. If the base of the triangle is 8 centimeters, make a table of values that shows the area of the triangle for heights 1, 2, 3, 4, 5, and 6 centimeters.
16. Make a table of values for the function $f(x) = \sqrt{2x+3}$ for the input values $-1, 0, 1, 2, 3, 4, 5$.
17. Write a function rule for the table.

x	3	4	5	6
y	9	16	25	36

18. Write a function rule for the table.

hours	0	1	2	3
cost	15	20	25	30

19. Write a function rule for the table.

x	0	1	2	3
y	24	12	6	3

20. Write a function that represents the number of cuts you need to cut a ribbon in x number of pieces.
21. Solomon charges a $40 flat rate and $25 per hour to repair a leaky pipe. Write a function that represents the total fee charged as a function of hours worked. How much does Solomon earn for a three-hour job?
22. Rochelle has invested $2500 in a jewelry making kit. She makes bracelets that she can sell for $12.50 each. How many bracelets does Rochelle need to make before she breaks even?
23. Make up a situation in which the domain is all real numbers but the range is all whole numbers.

Mixed Review

24. Compare the following numbers 23 ___ 21.999.
25. Write an equation to represent the following: the quotient of 96 and 4 is g.
26. Write an inequality to represent the following: 11 minus b is at least 77.
27. Find the value of the variable $k : 13(k) = 169$.

Quick Quiz

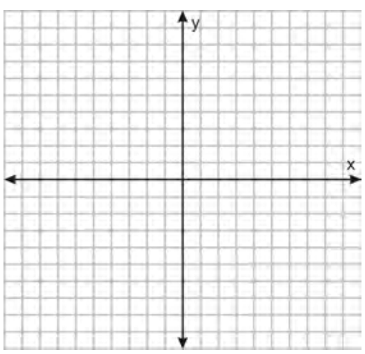
1. Write a function rule to describe the following table:

# of Books	1	2	3	4	5	6
Cost	4.75	5.25	5.75	6.25	6.75	7.25

2. Simplify: $84 \div [(18 - 16) \times 3]$.
3. Evaluate the expression $\frac{2}{3}(y + 6)$ when $y = 3$.
4. Rewrite using function notation: $y = \frac{1}{4}x^2$.
5. You purchased six video games for \$29.99 each and three DVD movies for \$22.99. What is the total amount of money you spent?

1.6 Functions as Graphs

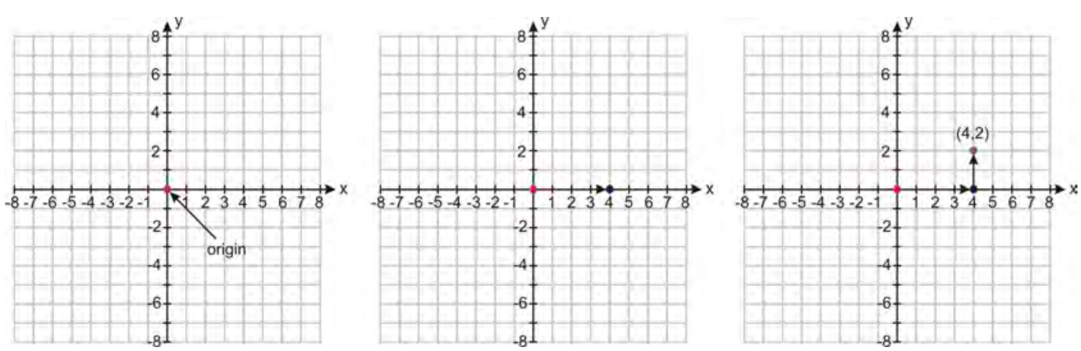
Once a table has been created for a function, the next step is to visualize the relationship by graphing the coordinates *(independent value, dependent value)*. In previous courses, you have learned how to plot ordered pairs on a coordinate plane. The first coordinate represents the horizontal distance from the origin (the point where the axes intersect). The second coordinate represents the vertical distance from the origin.



To graph a coordinate point such as (4,2) we start at the origin.

Because the first coordinate is positive four, we move 4 units to the right.

From this location, since the second coordinate is positive two, we move 2 units up.



Example 1: *Plot the following coordinate points on the Cartesian plane.*

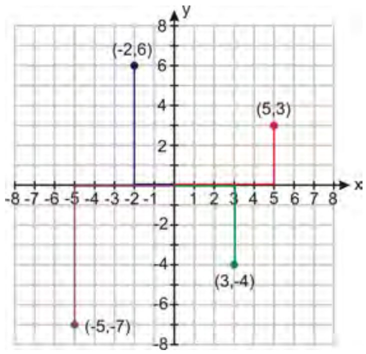
(a) (5, 3)

(b) (–2, 6)

(c) (3, –4)

(d) (–5, –7)

Solution: We show all the coordinate points on the same plot.



Notice that:

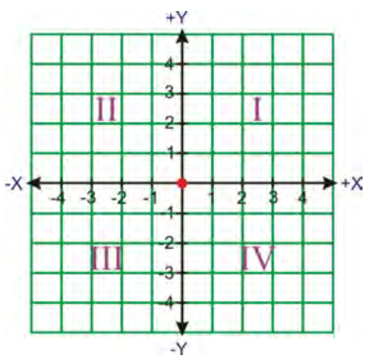
For a positive x value we move to the right.

For a negative x value we move to the left.

For a positive y value we move up.

For a negative y value we move down.

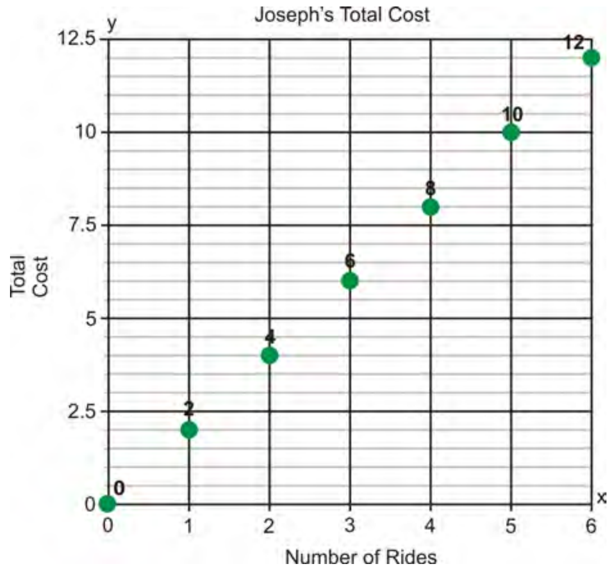
When referring to a coordinate plane, also called a Cartesian plane, the four sections are called **quadrants.** The first quadrant is the upper right section, the second quadrant is the upper left, the third quadrant is the lower left and the fourth quadrant is the lower right.



Suppose we wanted to visualize Joseph's total cost of riding at the amusement park. Using the table generated in Lesson 1.5, the graph can be constructed as (number of rides, total cost).

Table 1.3:

r	$J(r) = 2r$
0	$2(0) = 0$
1	$2(1) = 2$
2	$2(2) = 4$
3	$2(3) = 6$
4	$2(4) = 8$
5	$2(5) = 10$
6	$2(6) = 12$



The green dots represent the combination of $(r, J(r))$. The dots are not connected because the domain of this function is all whole numbers. By connecting the points we are indicating that all values between the ordered pairs are also solutions to this function. Can Joseph ride $2\frac{1}{2}$ rides? Of course not! Therefore, we leave this situation as a **scatter plot.**

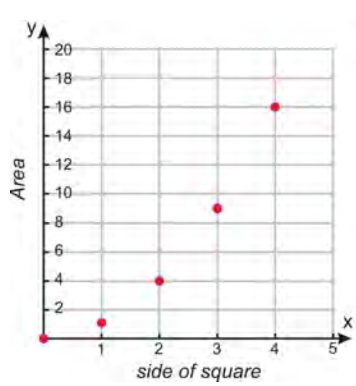
Example 2: Graph the function that has the following table of values.

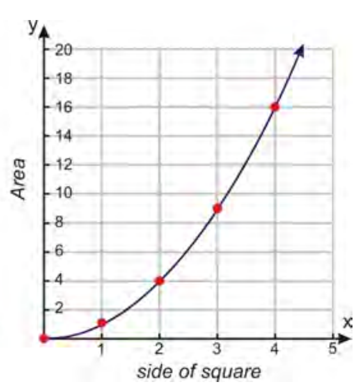
Side of the Square	0	1	2	3	4
Area of the Square	0	1	4	9	16

Solution: The table gives us five sets of coordinate points:

(0, 0), (1, 1), (2, 4), (3, 9), (4, 16).

To graph the function, we plot all the coordinate points. Because the length of a square can be fractional values, but not negative, the domain of this function is all positive real numbers, or $x \geq 0$. This means the ordered pairs can be connected with a smooth curve. This curve will continue forever in the positive direction, shown by an arrow.



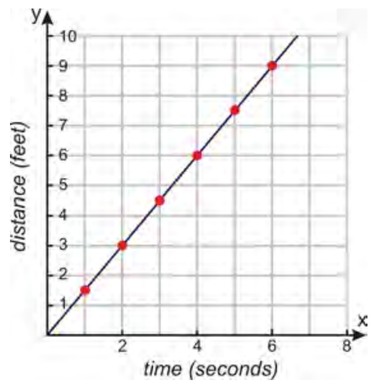


Writing a Function Rule Using a Graph

In many cases, you are given a graph and asked to determine its function. From a graph, you can read pairs of coordinate points that are on the curve of the function. The coordinate points give values of dependent and independent variables. These variables are related to each other by a rule. It is important we make sure this rule works for all the points on the curve.

In this course, you will learn to recognize different kinds of functions. There will be specific methods that you can use for each type of function that will help you find the function rule. For now, we will look at some basic examples and find patterns that will help us figure out the relationship between the dependent and independent variables.

Example 3: The graph below shows the distance that an inchworm covers over time. Find the function rule that shows how distance and time are related to each other.



Solution: Make table of values of several coordinate points to identify a pattern.

Time	0	1	2	3	4	5	6
Distance	0	1.5	3	4.5	6	7.5	9

We can see that for every minute the distance increases by 1.5 feet. We can write the function rule as:

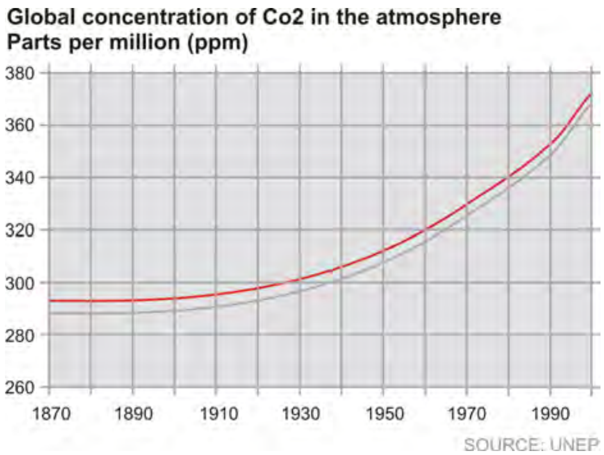
Distance $= 1.5 \times$ time

The equation of the function is $f(x) = 1.5x$

Analyze the Graph of a Real-World Situation

Graphs are used to represent data in all areas of life. You can find graphs in newspapers, political campaigns, science journals, and business presentations.

Here is an example of a graph you might see reported in the news. Most mainstream scientists believe that increased emissions of greenhouse gases, particularly carbon dioxide, are contributing to the warming of the planet. The graph below illustrates how carbon dioxide levels have increased as the world has industrialized.



From this graph, we can find the concentration of carbon dioxide found in the atmosphere in different years.

1900 - 285 parts per million

1930 - 300 parts per million

1950 - 310 parts per million

1990 - 350 parts per million

In Chapter 9, you will learn how to approximate an equation to fit this data using a graphing calculator.

Determining Whether a Relation Is a Function

You saw that a function is a **relation** between the independent and the dependent variables. It is a rule that uses the values of the independent variable to give the values of the dependent variable. A function rule can be expressed in words, as an equation, as a table of values, and as a graph. All representations are useful and necessary in understanding the relation between the variables.

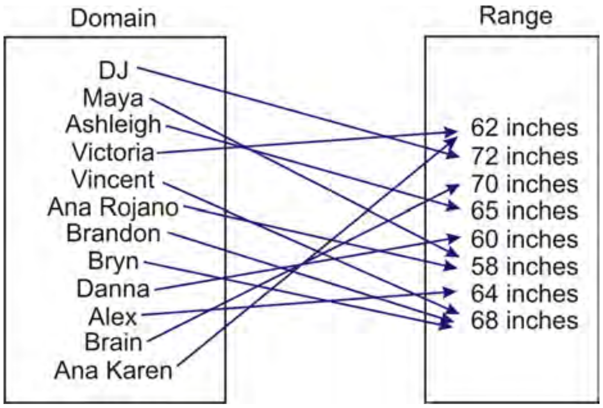
Definition: A **relation** is a set of ordered pairs.

Mathematically, a function is a special kind of relation.

Definition: A **function** is a relation between two variables such that the independent value has EXACTLY one dependent value.

This usually means that each $x-$value has only one $y-$value assigned to it. But, not all functions involve x and y.

Consider the relation that shows the heights of all students in a class. The domain is the set of people in the class and the range is the set of heights. Each person in the class cannot be more than one height at the same time. This relation is a function because for each person there is exactly one height that belongs to him or her.



Notice that in a function, a value in the range can belong to more than one element in the domain, so more than one person in the class can have the same height. The opposite is not possible, one person cannot have multiple heights.

Example 4: *Determine if the relation is a function.*

a) (1, 3), (–1, –2), (3, 5), (2, 5), (3, 4)

b) (–3, 20), (–5, 25), (–1, 5), (7, 12), (9, 2)

Solution:

a) To determine whether this relation is a function, we must follow the definition of a function. Each $x-$coordinate can have ONLY one $y-$coordinate. However, since the $x-$coordinate of 3 has two $y-$coordinates, 4 and 5, this relation is NOT a function.

b) Applying the definition of a function, each $x-$coordinate has only one $y-$coordinate. Therefore, this relation is a function.

Determining Whether a Graph Is a Function

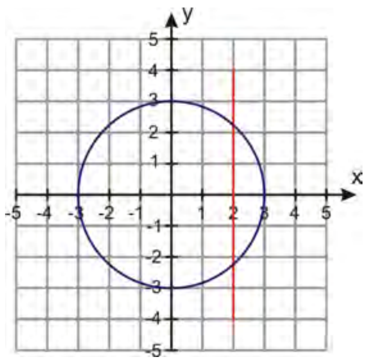
One way to determine whether a relation is a function is to construct a **flow chart** linking each dependent value to its matching independent value. Suppose, however, all you are given is the graph of the relation. How can you determine whether it is a function?

You could organize the ordered pairs into a table or a flow chart, similar to the student and height situation. This could be a lengthy process, but it is one possible way. A second way is to use the **Vertical Line Test.** Applying this test gives a quick and effective visual to decide if the graph is a function.

Theorem: Part A) A relation is a function if there are no vertical lines that intersect the graphed relation in more than one point.

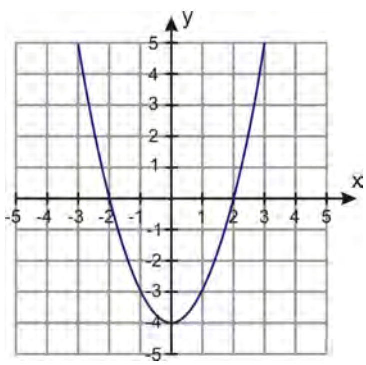
Part B) If a graphed relation does not intersect a vertical line in more than one point, then that relation is a function.

Is this graphed relation a function?



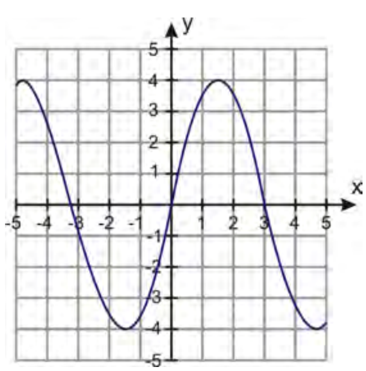
By drawing a vertical line (the red line) through the graph, we can see that the vertical line intersects the circle more than once. Therefore, this graph is NOT a function.

Here is a second example:



No matter where a vertical line is drawn through the graph, there will be only one intersection. Therefore, this graph is a function.

Example 4: *Determine if the relation is a function.*



Solution: Using the Vertical Line Test, we can conclude the relation is a function.

For more information:

Watch this YouTube video giving step-by-step instructions of the Vertical Line Test. CK-12 Basic Algebra: Vertical Line Test (3:11)

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the

Figure 1.8: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/715

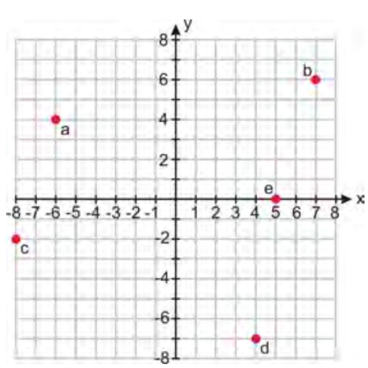
number of the practice exercise listed in the following exercise set.However, the practice exercise is the same in both. CK-12 Basic Algebra: Functions as Graphs (9:34)

Figure 1.9: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/716

In 1 – 5, plot the coordinate points on the Cartesian plane.

1. (4, –4)
2. (2, 7)
3. (–3, –5)
4. (6, 3)
5. (–4, 3)

Using the coordinate plane below, give the coordinates for a – e.

6. 

In 7 – 9, graph the relation on a coordinate plane. According to the situation, determine whether to connect the ordered pairs with a smooth curve or leave as a scatter plot.

7.

X	–10	–5	0	5	10
Y	–3	–0.5	2	4.5	7

8.

Table 1.4:

Side of cube (in inches)	**Volume of cube (in inches^3)**
0	0
1	1
2	8
3	27
4	64

9.

Table 1.5:

Time (in hours)	**Distance (in miles)**
–2	–50
–1	25
0	0
1	5
2	50

In 10 – 12, graph the function.

10. Brandon is a member of a movie club. He pays a \$50 annual membership and \$8 per movie.
11. $f(x) = (x - 2)^2$
12. $f(x) = 3.2^x$

In 13 – 16, determine if the relation is a function.

13. (1, 7), (2, 7), (3, 8), (4, 8), (5, 9)
14. (1, 1), (1, –1), (4, 2), (4, –2), (9, 3), (9, –3)
15.

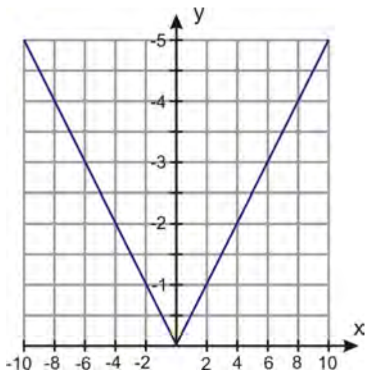
Age	20	25	25	30	35
Number of jobs by that age	3	4	7	4	2

16.

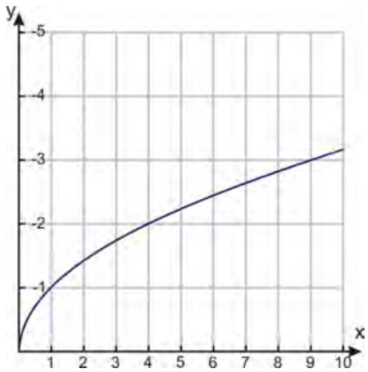
x	–4	–3	–2	–1	0
y	16	9	4	1	0

In 17 and 18, write a function rule for the graphed relation.

17.



18.



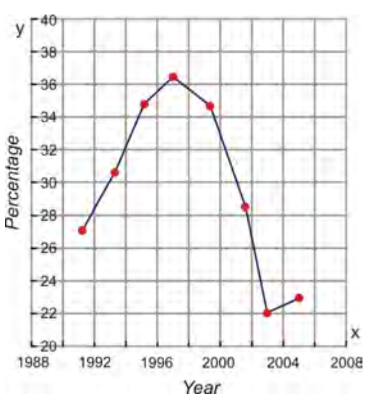
19. The students at a local high school took the Youth Risk Behavior Survey. The graph below shows the percentage of high school students who reported that they were current smokers. A person qualifies as a current smoker if he/she has smoked one or more cigarettes in the past 30 days. What percentage of high school students were current smokers in the following years?

(a) 1991

(b) 1996

(c) 2004

(d) 2005



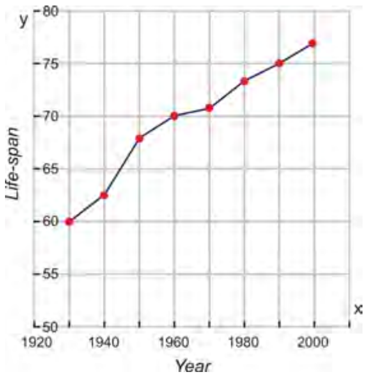
20. The graph below shows the average lifespan of people based on the year in which they were born. This information comes from the National Vital Statistics Report from the Center for Disease Control. What is the average lifespan of a person born in the following years?

(a) 1940

(b) 1955

(c) 1980

(d) 1995



21. The graph below shows the median income of an individual based on his/her number of years of education. The top curve shows the median income for males and the bottom curve shows the median income for females (Source: US Census, 2003). What is the median income of a male who has the following years of education?

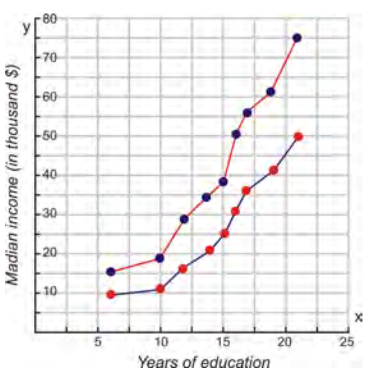
(a) 10 years of education

(b) 17 years of education

What is the median income of a female who has the same years of education?

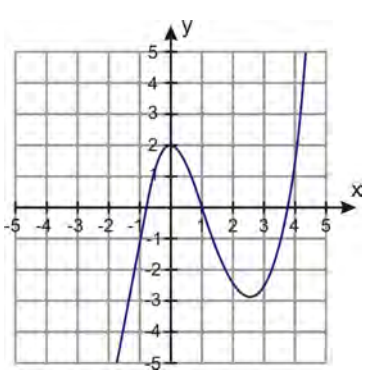
(c) 10 years of education

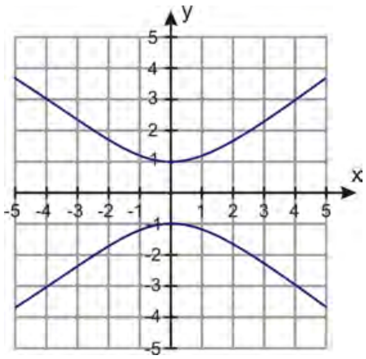
(d) 17 years of education



In 22 – 23, determine whether the graphed relation is a function.

22.



23. 

Mixed Review

24. A theme park charges \$12 entry to visitors. Find the money taken if 1296 people visit the park.
25. A group of students are in a room. After 25 students leave, it is found that $\frac{2}{3}$ of the original group are left in the room. How many students were in the room at the start?
26. Evaluate the expression: $\frac{x^2+9}{y+2}, y = 3 \ and \ x = 4$.
27. The amount of rubber needed to make a playground ball is found by the formula $A = 4\pi r^2$, where $r = radius$. Determine the amount of material needed to make a ball with a 7-inch radius.

1.7 A Problem-Solving Plan

Much of mathematics apply to real-world situations. To think critically and to problem solve are mathematical abilities. Although these capabilities may be the most challenging, they are also the most rewarding.

To be successful in applying mathematics in real-life situations, you must have a "toolbox" of strategies to assist you. The last few lessons of many chapters in this FlexBook are devoted to filling this toolbox so you to become a better problem solver and tackle mathematics in the real world.

Step #1: Read and Understand the Given Problem

Every problem you encounter gives you clues needed to solve it successfully. Here is a checklist you can use to help you understand the problem.

√ Read the problem carefully. Make sure you read all the sentences. Many mistakes have been made by failing to fully read the situation.

√ Underline or highlight key words. These include mathematical operations such as *sum, difference, product,* and mathematical verbs such as *equal, more than, less than, is.* Key words also include the nouns the situation is describing such as *time, distance, people,* etc.

Visit the Wylie Intermediate Website (http://wylie.region14.net/webs/shamilton/math_clue_words.htm) for more clue words.

√ Ask yourself if you have seen a problem like this before. Even though the nouns and verbs may be different, the general situation may be similar to something else you've seen.

√ What are you being asked to do? What is the question you are supposed to answer?

√ What facts are you given? These typically include numbers or other pieces of information.

Once you have discovered what the problem is about, the next step is to declare what variables will represent the nouns in the problem. Remember to use letters that make sense!

Step #2: Make a Plan to Solve the Problem

The next step in the problem-solving plan is to **make a plan** or **develop a strategy.** How can the information you know assist you in figuring out the unknown quantities?

Here are some common strategies that you will learn.

- Drawing a diagram
- Making a table
- Looking for a pattern
- Using guess and check
- Working backwards
- Using a formula
- Reading and making graphs
- Writing equations
- Using linear models
- Using dimensional analysis
- Using the right type of function for the situation

In most problems, you will use a combination of strategies. For example, drawing a diagram and looking for patterns are good strategies for most problems. Also, making a table and drawing a graph are often used together. The "writing an equation" strategy is the one you will work with the most frequently in your study of algebra.

Step #3: Solve the Problem and Check the Results

Once you develop a plan, you can use it to **solve the problem.**

The last step in solving any problem should always be to **check and interpret** the answer. Here are some questions to help you to do that.

- Does the answer make sense?
- If you substitute the solution into the original problem, does it make the sentence true?
- Can you use another method to arrive at the same answer?

Step #4: Compare Alternative Approaches

Sometimes a certain problem is best solved by using a specific method. Most of the time, however, it can be solved by using several different strategies. When you are familiar with all of the problem-solving strategies, it is up to you to choose the methods that you are most comfortable with and that make sense to you. In this book, we will often use more than one method to solve a problem. This way we can demonstrate the strengths and weaknesses of different strategies when applied to different types of problems.

Regardless of the strategy you are using, you should always implement the problem-solving plan when you are solving word problems. Here is a summary of the problem-solving plan.

Step 1: Understand the problem.

Step 2: Devise a plan – Translate. Come up with a way to solve the problem. Set up an equation, draw a diagram, make a chart, or construct a table as a start to begin your problem-solving plan.

Step 3: Carry out the plan – Solve.

Step 4: Check and Interpret: Check to see if you have used all your information. Then look to see if the answer makes sense.

Solve Real-World Problems Using a Plan

Example 1: Jeff is 10 years old. His younger brother, Ben, is 4 years old. How old will Jeff be when he is twice as old as Ben?

Solution: Begin by understanding the problem. Highlight the key words.

Jeff is **10** years old. His younger brother, **Ben**, is **4** years old. **How old** will Jeff be **when he is twice as old as Ben**?

The question we need to answer is. "What is Jeff's age when he is twice as old as Ben?"

You could guess and check, use a formula, make a table, or look for a pattern.

The key is "twice as old." This clue means two times, or double Ben's age. Begin by doubling possible ages. Let's look for a pattern.

$4 \times 2 = 8$. Jeff is already older than 8.

$5 \times 2 = 10$. This doesn't make sense because Jeff is already 10.

$6 \times 2 = 12$. In two years, Jeff will be 12 and Ben will be 6. Jeff will be twice as old.

Jeff will be 12 years old.

Example 2: Matthew is planning to harvest his corn crop this fall. The field has 660 rows of corn with 300 ears per row. Matthew estimates his crew will have the crop harvested in 20 hours. How many ears of corn will his crew harvest per hour?

Solution: Begin by highlighting the key information.

Matthew is planning to harvest his corn crop this fall. The field has **660 rows** of corn with **300 ears per row**. Matthew estimates his crew will have the **crop harvested in 20 hours. How many ears of corn** will his crew **harvest per hour**?

You could draw a picture (it may take a while), write an equation, look for a pattern, or make a table. Let's try to use reasoning.

We need to figure out how many ears of corn are in the field. $660(300) = 198,000$. This is how many ears are in the field. It will take 20 hours to harvest the entire field, so we need to divide 198,000 by 20 to get the number of ears picked per hour.

$$\frac{198,000}{20} = 9,900$$

The crew can harvest 9,900 ears per hour.

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set.However, the practice exercise is the same in both. CK-12 Basic Algebra: Word Problem-Solving Plan 1 (10:12)

Figure 1.10: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/719

1. What are the four steps to solving a problem?
2. Name three strategies you can use to help make a plan. Which one(s) are you most familiar with already?
3. Which types of strategies work well together? Why?
4. Suppose Matthew's crew takes 36 hours to harvest the field. How many ears per hour will they harvest?
5. Why is it difficult to solve Ben and Jeff's age problem by drawing a diagram?
6. How do you check a solution to a problem? What is the purpose of checking the solution?
7. There were 12 people on a jury, with four more women than men. How many women were there?
8. A rope 14 feet long is cut into two pieces. One piece is 2.25 feet longer than the other. What are the lengths of the two pieces?
9. A sweatshirt costs \$35. Find the total cost if the sales tax is 7.75%.
10. This year you got a 5% raise. If your new salary is \$45,000, what was your salary before the raise?
11. It costs \$250 to carpet a room that is $14\ ft \times 18\ ft$. How much does it cost to carpet a room that is $9\ ft \times 10\ ft$?
12. A department store has a 15% discount for employees. Suppose an employee has a coupon worth \$10 off any item and she wants to buy a \$65 purse. What is the final cost of the purse if the employee discount is applied before the coupon is subtracted?
13. To host a dance at a hotel, you must pay \$250 plus \$20 per guest. How much money would you have to pay for 25 guests?

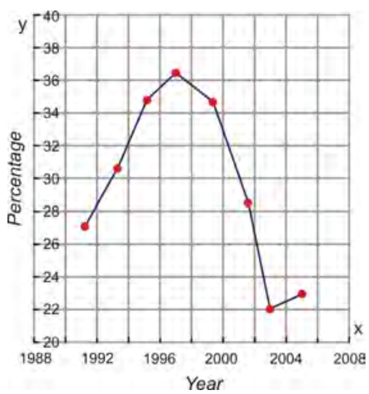
14. It costs \$12 to get into the San Diego County Fair and \$1.50 per ride. If Rena spent \$24 in total, how many rides did she go on?
15. An ice cream shop sells a small cone for \$2.92, a medium cone for \$3.50, and a large cone for \$4.25. Last Saturday, the shop sold 22 small cones, 26 medium cones, and 15 large cones. How much money did the store earn?
16. The sum of angles in a triangle is 180 degrees. If the second angle is twice the size of the first angle and the third angle is three times the size of the first angle, what are the measures of the angles in the triangle?

Mixed Review

17. Choose an appropriate variable for the following situation: *It takes Lily 45 minutes to bathe and groom a dog. How many dogs can she groom in an 9-hour day?*
18. Translate the following into an algebraic inequality: *Fourteen less than twice a number is greater than or equal to 16.*
19. Write the pattern of the table below in words and using an algebraic equation.

x	-2	-1	0	1
y	-8	-4	0	4

20. Check that $m = 4$ is a solution to $3y - 11 \geq -3$.
21. What is the domain and range of the graph below?



1.8 Problem-Solving Strategies: Make a Table; Look for a Pattern

This lesson focuses on two of the strategies introduced in the previous chapter: making a table and looking for a pattern. These are the most common strategies you have used before algebra. Let's review the four-step problem-solving plan from Lesson 1.7.

Step 1: Understand the problem.

Step 2: Devise a plan – Translate. Come up with a way to solve the problem. Set up an equation, draw a diagram, make a chart, or construct a table as a start to begin your problem-solving plan.

Step 3: Carry out the plan – Solve.

Step 4: Check and Interpret: Check to see if you used all your information. Then look to see if the answer makes sense.

Using a Table to Solve a Problem

When a problem has data that needs to be organized, a table is a highly effective problem-solving strategy. A table is also helpful when the problem asks you to record a large amount of information. Patterns and numerical relationships are easier to see when data are organized in a table.

Example 1: *Josie takes up jogging. In the first week she jogs for 10 minutes per day, in the second week she jogs for 12 minutes per day. Each week, she wants to increase her jogging time by 2 minutes per day. If she jogs six days per week each week, what will be her total jogging time in the sixth week?*

Solution: Organize the information in a table

Table 1.6:

Week 1	**Week 2**	**Week 3**	**Week 4**
10 minutes **60 min/week**	**12 minutes** **72 min/week**	**14 minutes** **84 min/week**	**16 minutes** **96 min/week**

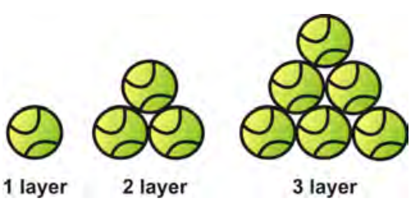
We can see the pattern that the number of minutes is increasing by 12 each week. Continuing this pattern, Josie will run 120 minutes in the sixth week.

Don't forget to check the solution! The pattern starts at 60 and adds 12 each week after the first week. The equation to represent this situation is $t = 60 + 12(w - 1)$. By substituting 6 for the variable of w, the equation becomes $t = 60 + 12(6 - 1) = 60 + 60 = 120$

Solve a Problem by Looking for a Pattern

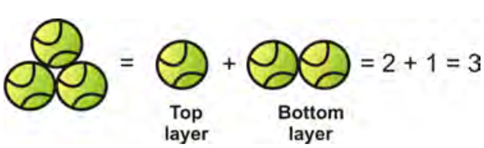
Some situations have a readily apparent pattern, which means that the pattern is easy to see. In this case, you may not need to organize the information into a table. Instead, you can use the pattern to arrive at your solution.

Example 2: *You arrange tennis balls in triangular shapes as shown. How many balls will there be in a triangle that has 8 layers?*

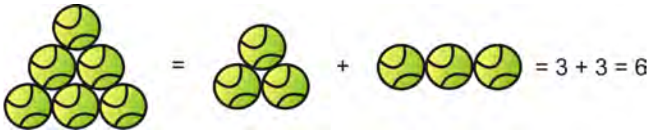


One layer: It is simple to see that a triangle with one layer has only one ball.

Two layers: For a triangle with two layers we add the balls from the top layer to the balls of the bottom layer. It is useful to make a sketch of the different layers in the triangle.



Three layers: we add the balls from the top triangle to the balls from the bottom layer.



We can fill the first three rows of the table.

1	2	3	4
1	3	6	$6+4=10$

To find the number of tennis balls in 8 layers, continue the pattern.

5	6	7	8
$10+5=15$	$15+6=21$	$21+7=28$	$28+8=36$

There will be 36 tennis balls in the 8 layers.

Check: Each layer of the triangle has one more ball than the previous one. In a triangle with 8 layers, each layer has the smae number of balls as its position. When we add these we get:

$1+2+3+4+5+6+7+8=36$ balls

The answer checks out.

Comparing Alternative Approaches to Solving Problems

In this section, we will compare the methods of "Making a Table" and "Looking for a Pattern" by using each method in turn to solve a problem.

Example 3: *Andrew cashes a \$180 check and wants the money in \$10 and \$20 bills. The bank teller gives him 12 bills. How many of each kind of bill does he receive?*

Solution: Method 1: Making a Table

Tens	0	2	4	6	8	10	12	14	16	18
Twenties	9	8	7	6	5	4	3	2	1	0

The combination that has a sum of 12 is six \$10 bills and six \$20 bills.

Method 2: Using a Pattern

The pattern is that for every pair of \$10 bills, the number of \$20 bills reduces by one. Begin with the most number of \$20 bills. For every \$20 bill lost, add two \$10 bills.

$$6(\$10)+6(\$20)=\$180$$

Check: Six \$10 bills and six \$20 bills $=6(\$10)+6(\$20)=\$60+\$120=\$180$.

Using These Strategies to Solve Problems

Example 4: *Students are going to march in a homecoming parade. There will be one kindergartener, two first-graders, three second-graders, and so on through* 12^{th} *grade. How many students will be walking in the homecoming parade?*

Could you make a table? Absolutely. Could you look for a pattern? Absolutely.

Solution 1: Make a table:

K	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13

The solution is the sum of all the numbers, 91. There will be 91 students walking in the homecoming parade.

Solution 2: Look for a pattern.

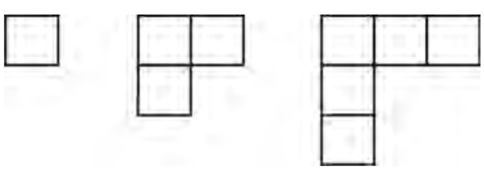
The pattern is: The number of students is one more than their grade level. Therefore, the solution is the sum of numbers from 1 (kindergarten) through 13 (12^{th} grade). The solution is 91.

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set.However, the practice exercise is the same in both. CK-12 Basic Algebra: Word Problem-Solving Strategies (12:51)

Figure 1.11: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/722

1. Go back and find the solution to the problem in Example 1.
2. Britt has $2.25 in nickels and dimes. If she has 40 coins in total how many of each coin does she have?
3. A pattern of squares is placed together as shown. How many squares are in the 12^{th} diagram?



4. Oswald is trying to cut down on drinking coffee. His goal is to cut down to 6 cups per week. If he starts with 24 cups the first week, cuts down to 21 cups the second week, and drops to 18 cups the third week, how many weeks will it take him to reach his goal?
5. Taylor checked out a book from the library and it is now 5 days late. The late fee is 10 cents per day. How much is the fine?
6. How many hours will a car traveling at 75 miles per hour take to catch up to a car traveling at 55 miles per hour if the slower car starts two hours before the faster car?
7. Grace starts biking at 12 miles per hour. One hour later, Dan starts biking at 15 miles per hour, following the same route. How long would it take him to catch up with Grace?

8. Lemuel wants to enclose a rectangular plot of land with a fence. He has 24 feet of fencing. What is the largest possible area that he could enclose with the fence?

Mixed Review

9. Determine if the relation is a function: $\{(2,6),(-9,0),(7,7),(3,5),(5,3)\}$.
10. Roy works construction during the summer and earns $78 per job. Create a table relating the number of jobs he could work, j, and the total amount of money he can earn, m.
11. Graph the following order pairs: (4,4); (–5,6), (–1,–1), (–7,–9), (2,–5)
12. Evaluate the following expression: $-4(4z-x+5)$; use $x=-10$, and $z=-8$.
13. The area of a circle is given by the formula $A=\pi r^2$. Determine the area of a circle with radius 6 mm.
14. Louie bought 9 packs of gum at $1.19 each. How much money did he spend?
15. Write the following without the multiplication symbol: $16 \times \frac{1}{8}c$.

1.9 Chapter 1 Review

Define the following words:

1. Domain
2. Range
3. Solution
4. Evaluate
5. Substitute
6. Operation
7. Variable
8. Algebraic expression
9. Equation
10. Algebraic inequality
11. Function
12. Independent variable

Evaluate the following expressions.

13. $3y(7-(z-y))$; use $y=-7$ and $z=2$
14. $\frac{m+3n-p}{4}$; use $m=9,\ n=7$, and $p=2$
15. $|p|-\left(\frac{n}{2}\right)^3$; use $n=2$ and $p=3$
16. $|v-21|,\ v=-70$

Choose an appropriate variable to describe the situation.

17. The number of candies you can eat in a day
18. The number of tomatoes a plant can grow
19. The number of cats at a humane society
20. The amount of snow on the ground
21. The number of water skiers on a lake
22. The number of geese migrating south

23. The number of people at a trade show

The surface area of a sphere is found by the formula $A = 4\pi r^2$. Determine the surface area for the following radii/diameters.

24. $radius = 10\ inches$
25. $radius = 2.4\ cm$
26. $diameter = 19\ meters$
27. $radius = 0.98\ mm$
28. $diameter = 5.5\ inches$

Insert parentheses to make a true equation.

29. $1 + 2 \cdot 3 + 4 = 15$
30. $5 \cdot 3 - 2 + 6 = 35$
31. $3 + 1 \cdot 7 - 2^2 \cdot 9 - 7 = 24$
32. $4 + 6 \cdot 2 \cdot 5 - 3 = 40$
33. $3^2 + 2 \cdot 7 - 4 = 33$

Translate the following into an algebraic equation or inequality.

34. Thirty-seven more than a number is 612.
35. The product of u and –7 equals 343.
36. The quotient of k and 18
37. Eleven less than a number is 43.
38. A number divided by –9 is –78.
39. The difference between 8 and h is 25.
40. The product of 8, –2, and r
41. Four plus m is less than or equal to 19.
42. Six is less than c.
43. Forty-two less than y is greater than 57.

Write the pattern shown in the table with words and with an algebraic equation.

44.

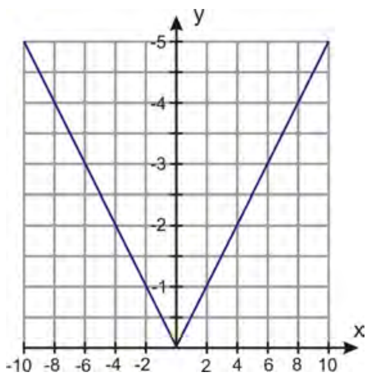
Movies watched	0	1	2	3	4	5
Total time	0	1.5	3	4.5	6	7.5

45. A case of donuts is sold by the half-dozen. Suppose 168 people purchase cases of donuts. How many individual donuts have been sold?
46. Write an inequality to represent the situation: *Peter's Lawn Mowing Service charges $10 per mowing job and $35 per landscaping job. Peter earns at least $8,600 each summer.*

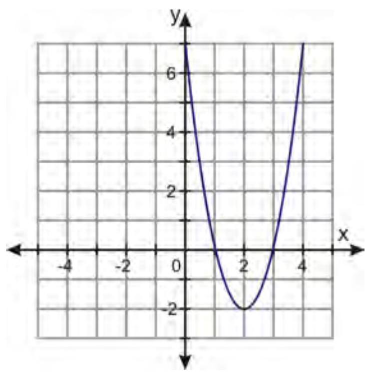
Check that the given number is a solution to the given equation or inequality.

47. $t = 0.9,\ 54 \le 7(9t + 5)$
48. $f = 2;\ f + 2 + 5f = 14$
49. $p = -6;\ 4p - 5p \le 5$

50. Logan has a cell phone service that charges \$18 dollars per month and \$0.05 per text message. Represent Logan's monthly cost as a function of the number of texts he sends per month.
51. An online video club charges \$14.99 per month. Represent the total cost of the video club as a function of the number of months that someone has been a member.
52. What is the domain and range for the following graph?



53. Henry invested \$5,100 in a vending machine service. Each machine pays him \$128. How many machines does Henry need to install to break even?
54. Is the following relation a function?

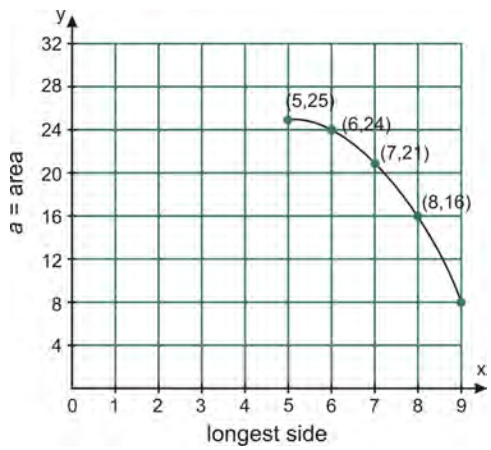


Solve the following questions using the 4-step problem-solving plan.

55. Together, the Raccoons and the Pelicans won 38 games. If the Raccoons won 13 games, how many games did the Pelicans win?
56. Elmville has 250 fewer people than Maplewood. Elmville has 900 people. How many people live in Maplewood?
57. The cell phone Bonus Plan gives you 4 times as many minutes as the Basic Plan. The Bonus Plan gives you a total of 1200 minutes. How many minutes does the Basic Plan give?
58. Margarite exercised for 24 minutes each day for a week. How many total minutes did Margarite exercise?
59. The downtown theater costs \$1.50 less than the mall theater. Each ticket at the downtown theater costs \$8. How much do tickets at the mall theater cost?
60. Mega Tape has 75 more feet of tape than everyday tape. A roll of Mega Tape has 225 feet of tape. How many feet does everyday tape have?
61. In bowling DeWayne got 3.5 times as many strikes as Junior. If DeWayne got 28 strikes, how many strikes did Junior get?

1.10 Chapter 1 Test

1. Write the following as an algebraic equation and determine its value. *On the stock market, Global First hit a price of $255 on Wednesday. This was $59 greater than the price on Tuesday. What was the price on Tuesday?*
2. The oak tree is 40 feet taller than the maple. Write an expression that represents the height of the oak.
3. Graph the following ordered pairs: (1, 2), (2, 3), (3, 4), (4, 5) (5, 6), (6, 7).
4. Determine the domain and range of the following function:



5. Is the following relation a function? Explain your answer. $\{(3,2),(3,4),(5,6),(7,8)\}$
6. Evaluate the expression if $a = 2,\ b = 3,\ c = 4;\ (5bc) - a$.
7. Simplify: $3[36 \div (3+6)]$.
8. Translate into an algebraic equation and find the value of the variable. *One-eighth of a pizza costs $1.09. How much was the entire pizza?*
9. Use the 4-step problem-solving method to determine the solution: *The freshman class has 17 more girls than boys. There are 561 freshmen. How many are girls?*
10. Underline the math verb in this sentence: The quotient of 8 and y is 48.
11. Jesse packs 16 boxes per hour. Complete the table to represent this situation.

Hours	0	2	4	5	8	10	12	14
Boxes								

12. A group of students are in a room. After 18 leave, it is found that $\frac{7}{8}$ of the original number of students remain. How many students were in the room in the beginning?
13. What are the domain and range of the following relation: $\{(2,3),(4,5),(6,7),(-2,-3),(-3,-4)\}$?
14. Write a function rule for the table:

Time in hours, x	0	1	2	3	4
Distance in miles, y	0	60	120	180	240

15. Determine if the given number is a solution to the inequality: $\frac{6-y}{y} > -8;\ y = 6$

Image Sources

(1) http://www.publicdomainpictures.net/view-image.php?image=1559&picture=christmas-baubles.